Rigorous justification of the effective boundary condition on a porous wall

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Abstract. Porous boundaries appear naturally in many applications. Concrete, wood, stone, soil and most biological tissues (including human skin) are porous materials. If their porosity is large enough to be taken into account in the model, then we need an appropriate effective boundary condition. The goal of our work is to derive rigorously the new effective boundary condition for the fluid flow in domain with porous boundary. Starting from the Newtonian fluid flow through a domain with an array of small holes on the boundary, using the homogenization and the boundary layers, we find an effective law in the form of generalized Darcy law. The effective law states that the velocity on the boundary is proportional to the difference between the interior and the exterior stresses on the boundary. That law describes, not only the leaking/perspiration through the boundary (the normal velocity) but also the slipping of the fluid in direction tangential to the boundary. The proportionality is given by a symmetric positive tensor, corresponding to the permeability of the boundary and depending on the form and the distribution of the pores. If the pores geometry is isotropic, then the condition splits in Beavers–Joseph type condition for the tangential flow and the standard Darcy condition for the normal flow.

Keywords: Porous boundary; viscous fluid; homogenization; Darcy-type boundary condition; boundary layers.

References

[1] E. Marušić-Paloka, I. Pažanin. *Rigorous justification of the effective boundary condition on a porous wall via homogenization*. Zeitschrift für Angewandte Mathematik und Physik, 72 (2021), 146.

[2] E.Marušić-Paloka, I.Pažanin. *The effective boundary condition on a porous wall*, International Journal of Engineering Science 173 (2022) 103638.

[3] E.Marušić-Paloka, I.Pažanin. *The effect of roughness on the boundary condition on porous wall*, Zeitschrift fÃ$\frac{1}{4}$r angewandte Mathematik und Physik, 74 (2023) 4; 129, 11.

[4] E.Marušić-Paloka. *Rigorous derivation of the Darcy boundary condition on the porous wall*, Mathematical methods in the applied sciences, 46 (2023) 9, 11021-11042.